

## Instructions

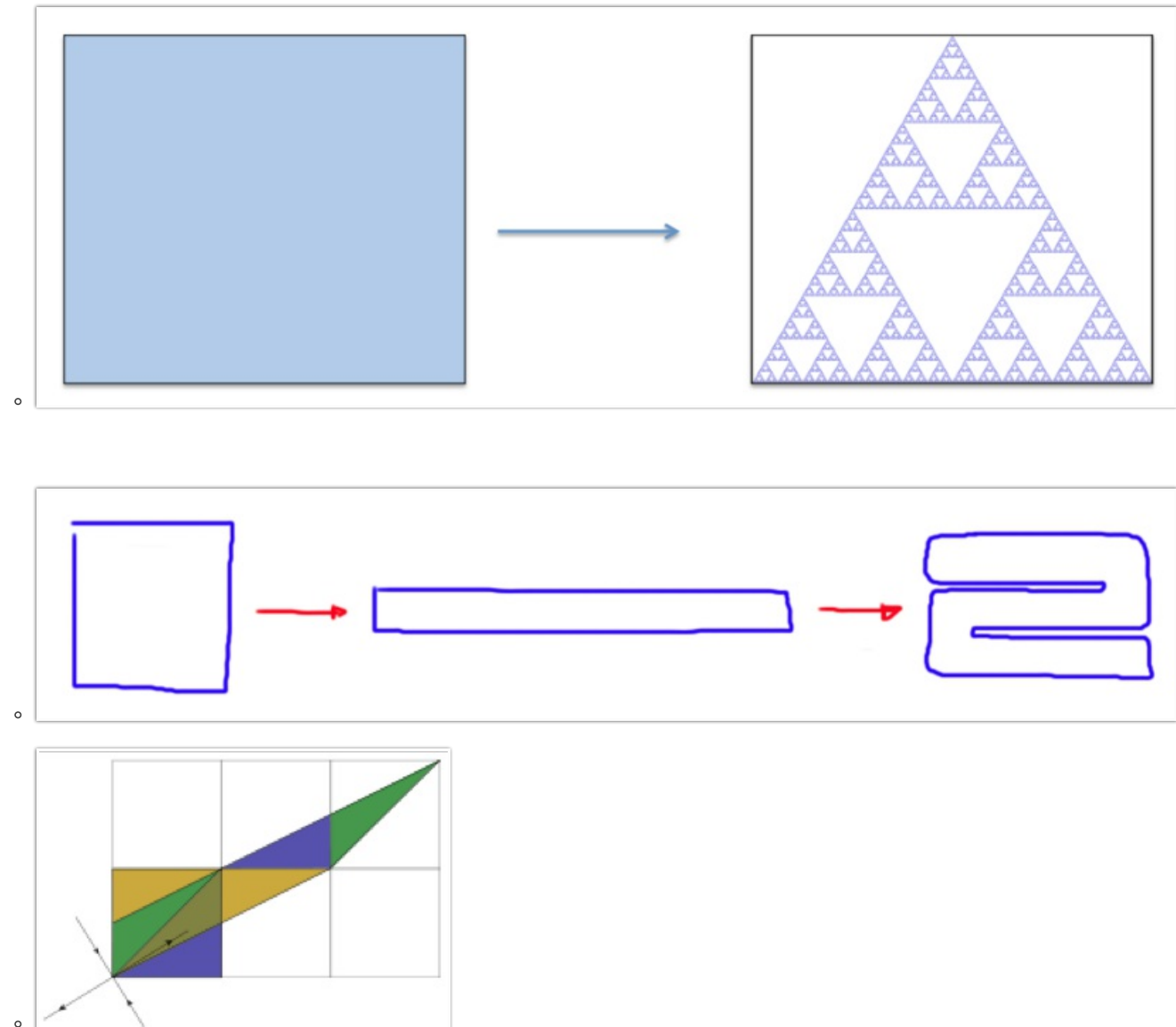
You may use any course materials, websites, books, computer programs, calculators, etc. for this test. Just don't ask another person for the answers or share your answers with other people. Be aware that simply typing the question text into google is unlikely to get you directly to the right answer; you're going to have to read what you find there in order to extract that answer, and the course videos are probably a faster way to do that.

"Experts" notes clarify situations that haven't been covered in this course, but that may introduce subtleties into the exam answers. Do not worry about them unless you understand the terms and issues in those notes.

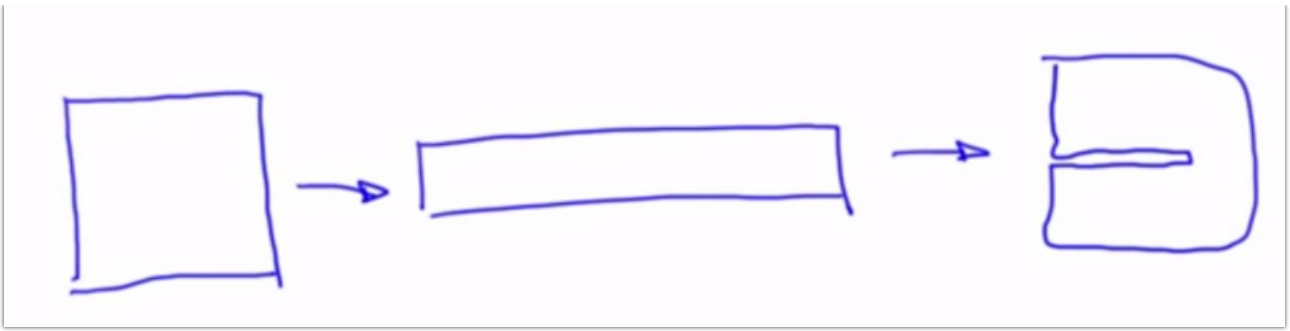
If you have questions about this test, please email us at [nonlinear@complexityexplorer.org](mailto:nonlinear@complexityexplorer.org) rather than posting on the forum.

## Question 1

Which of these is the Smale's horseshoe map? (Experts: please select the original horseshoe map *and not* any variation of it.)

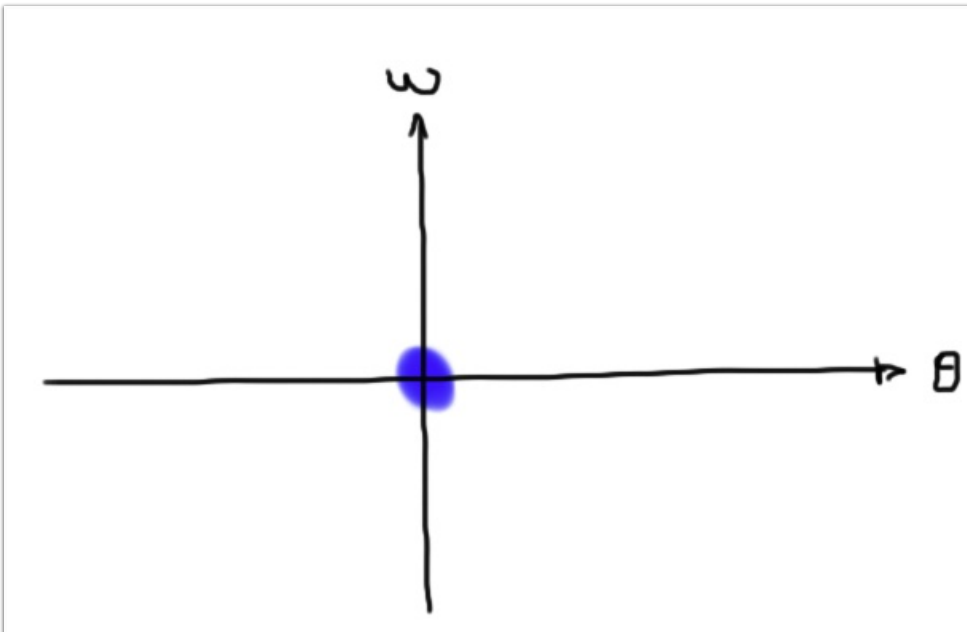
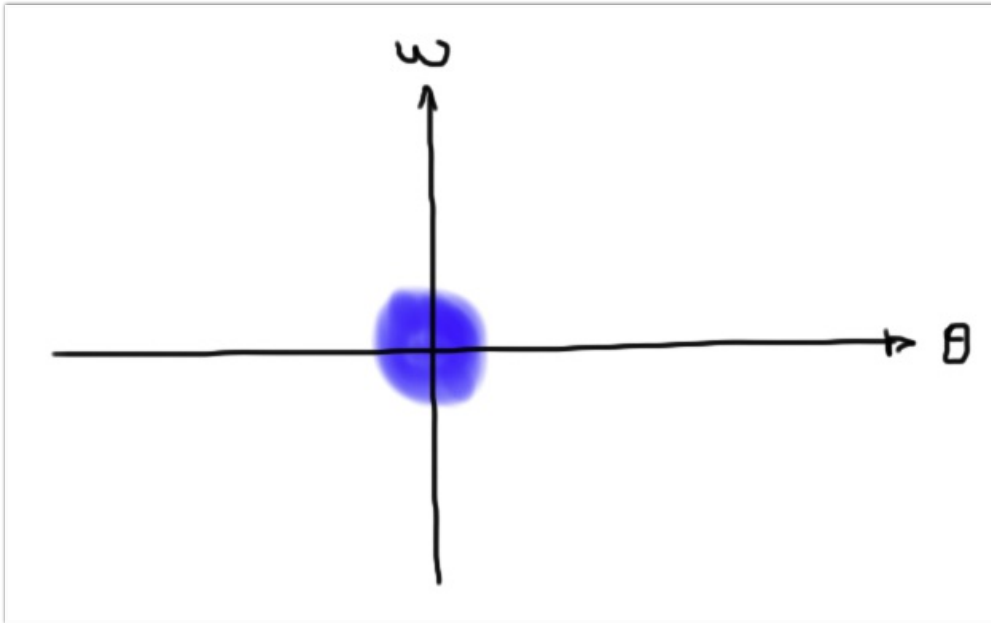


Note: the image above shows the unit square at the bottom left, then the deformed version stretched up and to the right. That deformed region then gets mapped back down onto the unit square in a manner depicted by the color coding (i.e., the green region maps to the green region and so on).

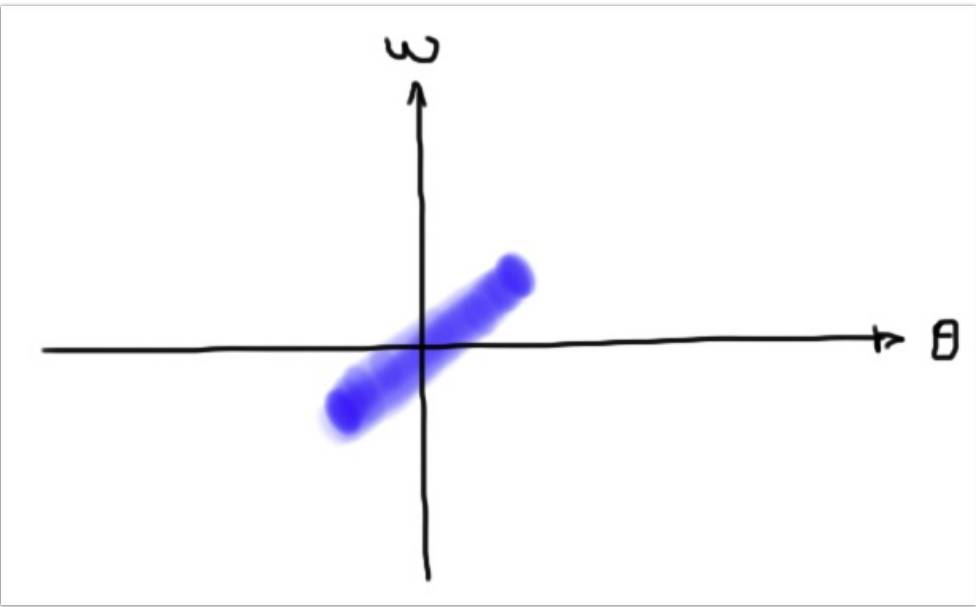
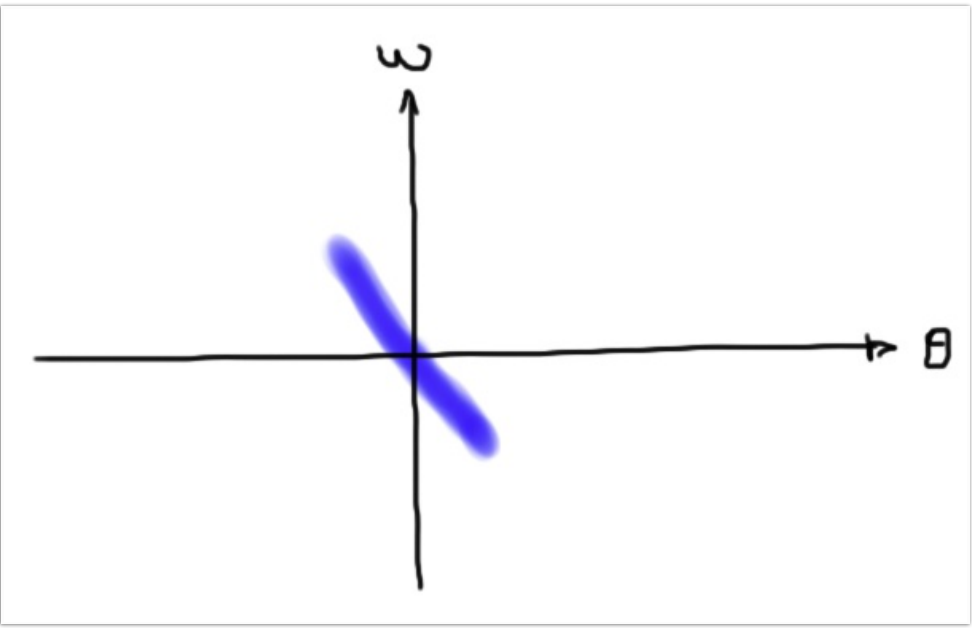
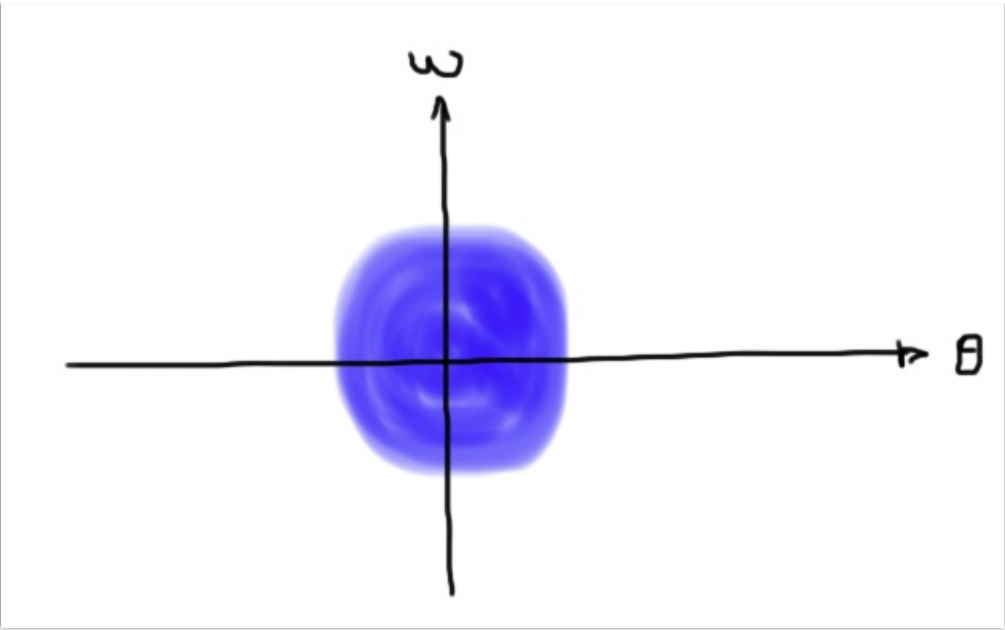


**Question 2**

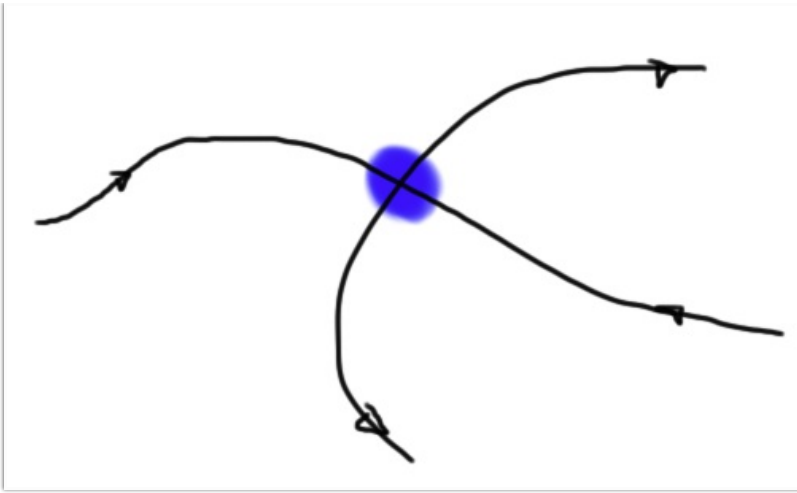
Which of the pictures below shows the shape of the blue ball of initial conditions evolved forward in time under the influence of the damped pendulum dynamics?



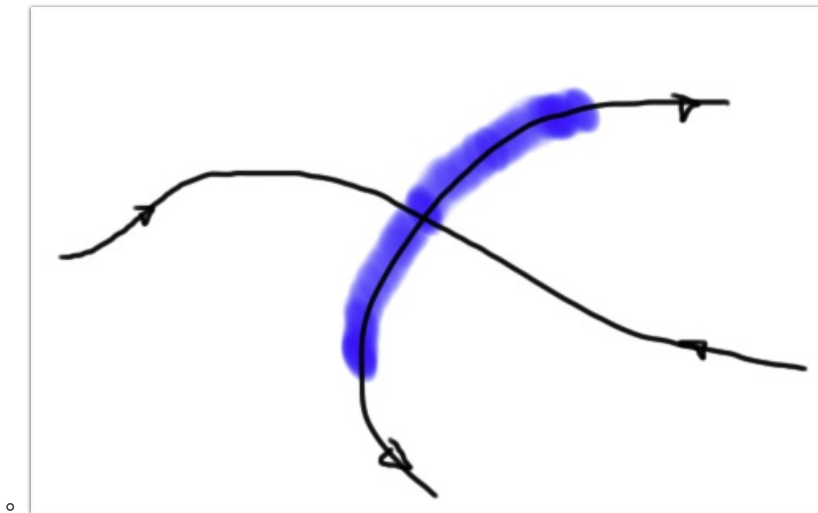
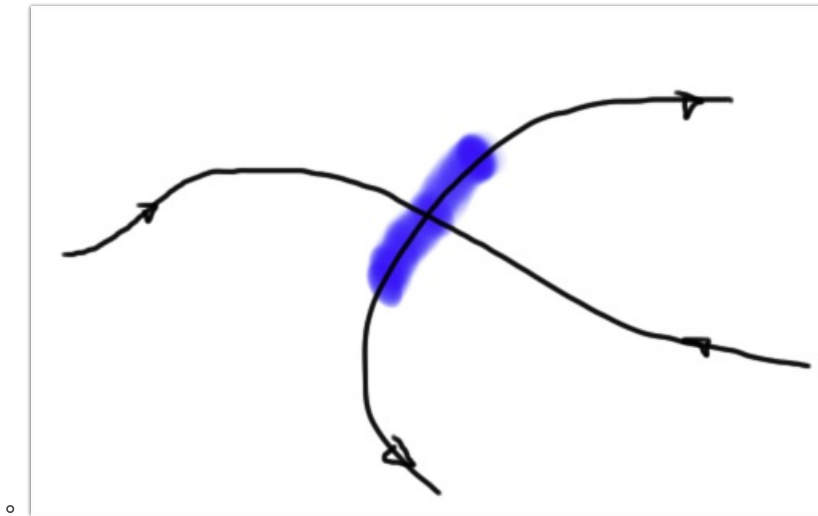
o

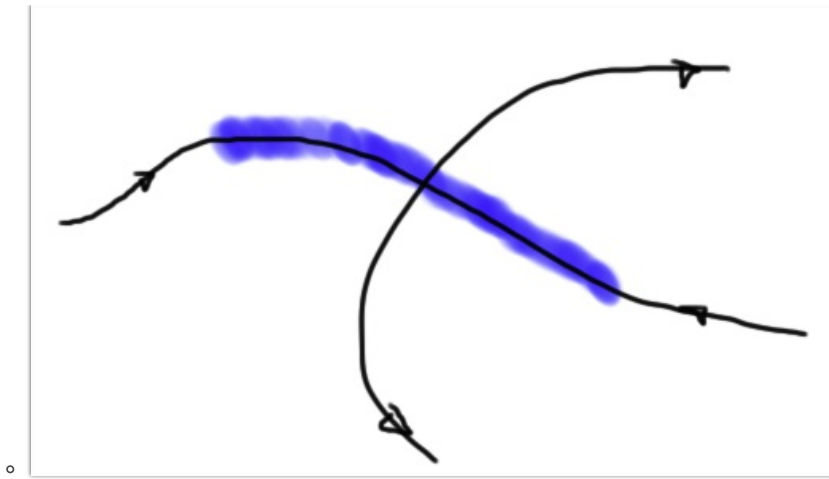


Question 3



If the positive  $\lambda$  is much bigger than the negative  $\lambda$ , which of the pictures below accurately reflects where the blue ball of initial conditions will go under the influence of the dynamics?





#### Question 4

The action of a dissipative dynamical system contracts the state space *everywhere*.

In this question, and the ones that follow, please use the specific restricted definition of dissipation that was given in the course videos.

- True
- False

#### Question 5

The action of a nondissipative dynamical system preserves the state space volume everywhere.

- True
- False

#### Question 6

Each stable and unstable manifold in a dynamical system has (Experts: at least) one associated Lyapunov exponent.

- True
- False

#### Question 7

Each unstable manifold in a dynamical system has (Experts: at least) one associated Lyapunov exponent whose value is negative.

- True
- False

#### Question 8

In a dissipative system, the sum of the Lyapunov exponents must be negative.

- True
- False

#### Question 9

In a dissipative system, there must be at least one negative Lyapunov exponent.

- True

- False

#### Question 10

In a dissipative system, there cannot be *any* positive Lyapunov exponents.

- True
- False

#### Question 11

A system with a chaotic attractor has at least one positive  $\lambda$  and at least one negative  $\lambda$ .

- True
- False

#### Question 12

Why are projections and sections useful?

- A. Because they reduce dimension.
- B. Because they make it easier to see the period of a periodic orbit.
- C. Because they make it easier to see bifurcations.
- D. All of A-C above.
- E. None of A-C above.
- F. Only some of A-C above.

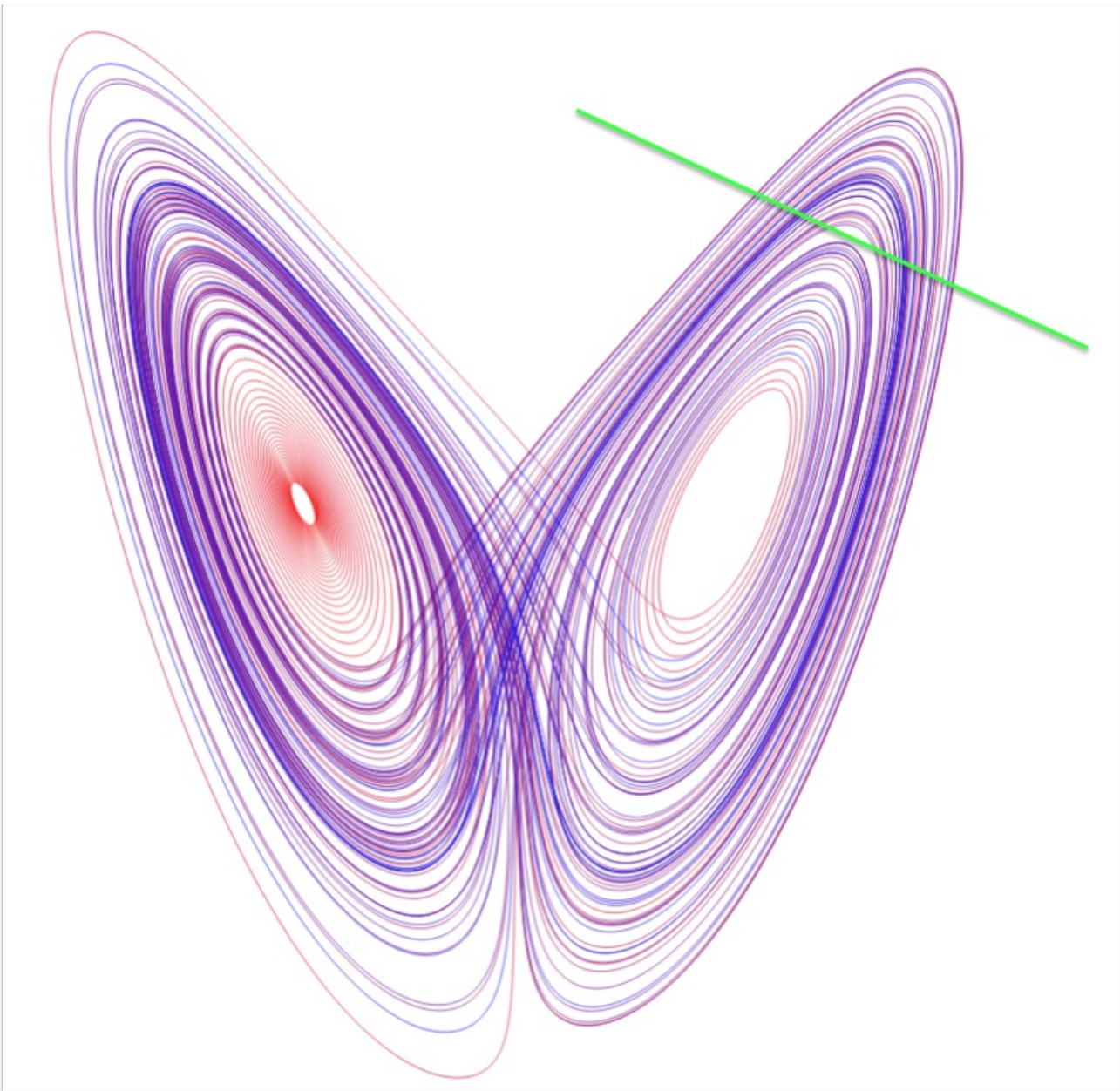
#### Question 13

Constructing temporal sections — e.g., with a strobe light — discretizes time, converting a flow into a map.

- True
- False

#### Question 14

If we section the Lorenz attractor (seen in the figure) across the green line, what will we see?



- A straight line (if the trajectory is infinitely long).
- A copy of the Logistic bifurcation diagram.
- A Cantor set.
- A copy of the Lorenz attractor.

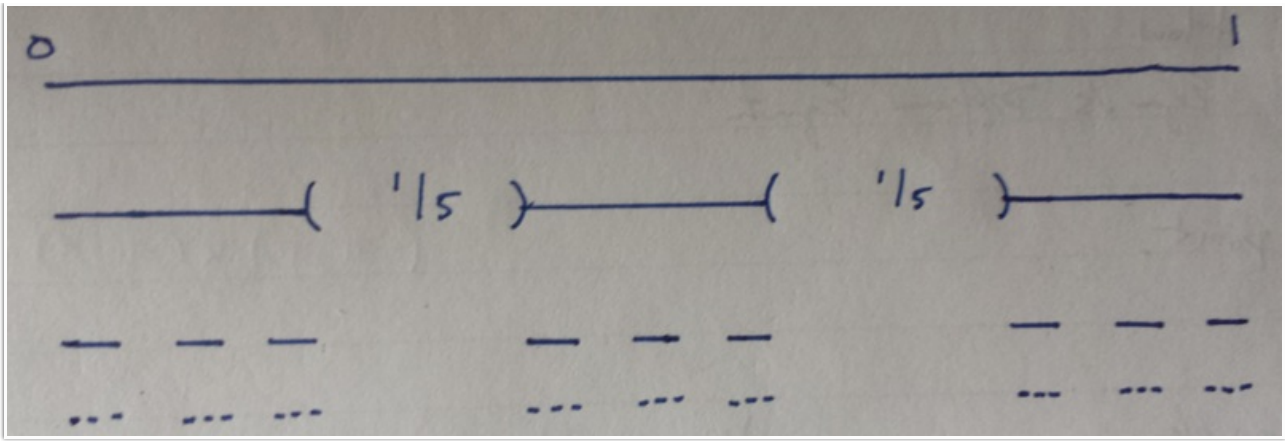
**Question 15**

What is the capacity dimension of the unit line segment?

- 0.63
- 0
- 1
- 0.876

**Question 16**

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What is the capacity dimension of this cantor set (which we will call  $C_{2/5}$ )?

- 0.6309
- 0.8614
- 0.6826
- 0.4572

#### Question 17

Let  $C_{2/5}$  to be the Cantor set defined in Question 16, and define  $C_{1/5}$  and  $C_{1/3}$  to be the middle-fifth- and middle-third-removed Cantor set respectively. Finally define " $\leftarrow$ " to be the "less than" operator on how much space an object fills as approximated by capacity dimension, (e.g., a point  $\leftarrow$  a line, by this definition). Which of the following string of inequalities is true?

- $C_{2/5} < C_{1/3} < C_{1/5}$
- $C_{1/3} < C_{2/5} < C_{1/5}$
- $C_{1/5} < C_{1/3} < C_{2/5}$
- $C_{1/5} < C_{2/5} < C_{1/3}$