

## Instructions

You may use any course materials, websites, books, computer programs, calculators, etc. for this test. Just don't ask another person for the answers or share your answers with other people. Be aware that simply typing the question text into google is unlikely to get you directly to the right answer; you're going to have to read what you find there in order to extract that answer, and the course videos are probably a faster way to do that.

"Experts" notes clarify situations that haven't been covered in this course, but that may introduce subtleties into the exam answers. Do not worry about them unless you understand the terms and issues in those notes.

If you have questions about this test, please email us at [nonlinear@complexityexplorer.org](mailto:nonlinear@complexityexplorer.org) rather than posting on the forum.

### Question 1

Numerical dynamics—the extra effects added to the true trajectory of a dynamical system by the ODE solver that you use to generate that trajectory—are affected by the time step.

- True
- False

### Question 2

Numerical dynamics, as defined above, are affected by the solver method (e.g., whether you use forward or backward Euler).

- True
- False

### Question 3

Numerical dynamics (as defined above) are obvious and easy to spot.

- True
- False

### Question 4

The effects produced by numerical dynamics can look like real, physical effects.

- True
- False

### Question 5

Dynamical error can "snowball" over the course of the solution of an ODE.

- True
- False

### Question 6

Which of the following is a method to derive the mathematical form of the local (one-step) truncation error of the forward Euler method?

- By running two nearby trajectories and watching the distance between them.
- By using a Taylor series.
- By requesting divine intervention.

### Question 7

What is machine epsilon?

- A way to describe the **smallest** positive number that a computer can store.
- A way to describe the **largest** positive number that a computer can store.
- The clock rate of the computer's processor.
- The name for the portion of the truncation-error term related to the stepsize, e.g.,  $(\Delta x)^2$  for forward Euler.

### Question 8

Why should we care about machine epsilon?

- Because it can introduce truncation error into computations.
- Because it can introduce roundoff error into computations.
- Because it can introduce observational error into computations.

### Question 9

How does the trapezoidal method for solving ODEs work, loosely speaking?

- It is an adaptive version of forward Euler.
- It averages a forward and backward Euler step to approximate each step
- It is a symplectic method.

### Question 10

Why is it a good idea to adapt the time step of an ODE solver 'on the fly'?

- Because dynamical landscapes can be highly heterogeneous (different shapes in different regions).
- Because using a tiny time step in a dynamically smooth region can be overkill.
- Because using a large time step in a dynamically complex region can cause errors.
- All of the above.
- None of the above.

### Question 11

Which of these strategies is a good way to adapt the time step of an ODE solver?

- Use different-size time steps and adapt the time step if the results change.
- Use a different ODE and adapt the time step if the results change.
- Generate a longer trajectory and adapt the time step if the endpoint is different.

### Question 12

The trapezoidal method is a member of the RK family of ODE solvers.

- True
- False

### Question 13

What should you do in order to increase your confidence that an ODE solver is giving you a good answer?

- Change the time step and see if that changes the results.
- Change the arithmetic precision and see if that changes the results.
- Change the method and see if that changes the results.
- All of the above.
- None of the above.

#### Question 14

Can you ever *prove* that an ODE solver is giving you the correct answer when you apply it to a chaotic ODE system?

- Yes
- No

#### Question 15

The shadowing lemma tells us that small changes in the parameters of a chaotic dynamical system bump the trajectory onto a "shadow trajectory" — an attractor thread that you'd have gotten to eventually anyway (in forward or backward time).

- True
- False

#### Question 16

Use your trapezoidal solver from HW 6.3 on the SHO equations with  $k=2$ ,  $m=0.5$ , and  $\beta=0$ , from the initial condition  $x(t=0)=-1$ ,  $v(t=0)=-2$ , with a timestep of 0.05, to compute the values of  $x$  and  $v$  at  $t=0.5$ . [Note: this is like problem 1 on HW 6.3, but with a different  $h$ ]

- $x(t = 0.5) \approx -1.6793, v(t = 0.5) \approx -0.6004$
- $x(t = 0.5) \approx -1.3814, v(t = 0.5) \approx 0.6070$
- $x(t = 0.5) \approx -1.3811, v(t = 0.5) \approx 0.6211$