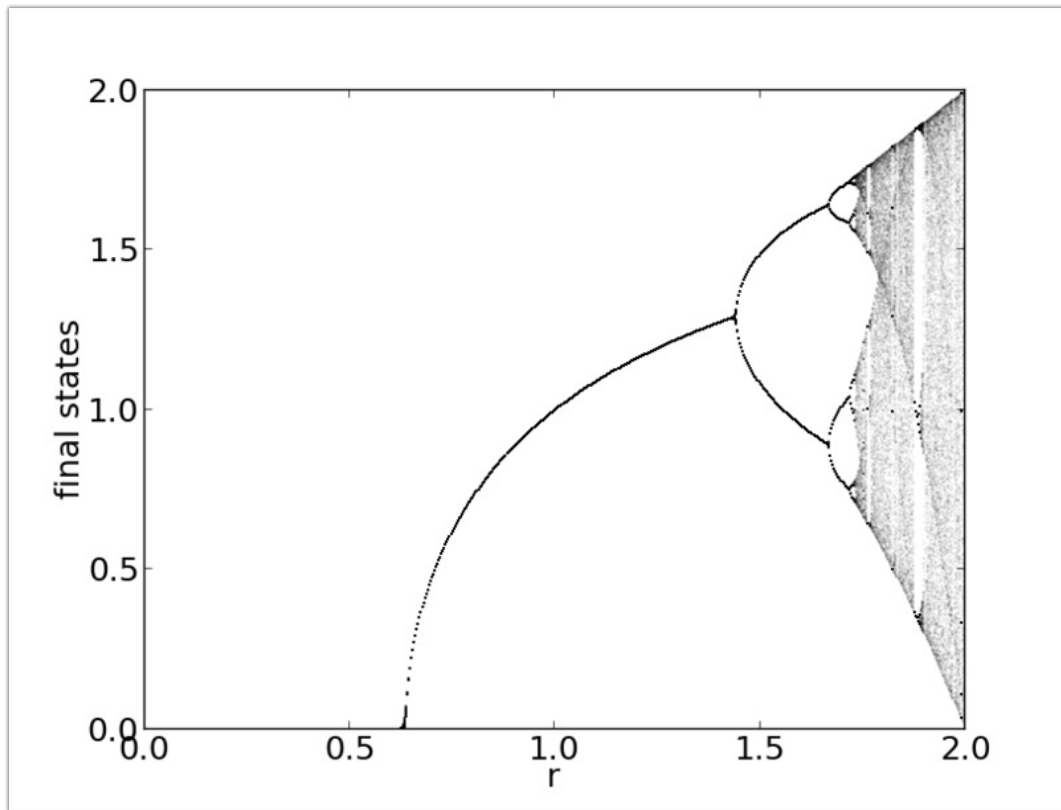


## Instructions

You may use any course materials, videos, websites, calculators, etc. for this test. Just don't ask another person for the answers or share your answers with other people. Please do not post questions about the test on the forum. If you have questions, please send them via email to [chaos@complexityexplorer.org](mailto:chaos@complexityexplorer.org). Thanks.

### Question 1



The bifurcation diagram for an iterated function (not the logistic equation) is shown in the figure above. Which statement best describes the long-term behavior of orbits of this dynamical system if  $r=1.0$ ?

- The orbit approaches zero
- The orbit approaches a fixed point near  $x = 1$ .
- The orbit approaches a fixed point near  $x = 1.3$ .
- The orbit is periodic with period 2
- The orbit appears to be aperiodic.

### Question 2

The bifurcation diagram for an iterated function (not the logistic equation) is shown in Question 1. Which statement best describes the long-term behavior of orbits of this dynamical system if  $r=1.5$ ?

- The orbit approaches zero
- The orbit approaches a fixed point near  $x = 1$ .
- The orbit approaches a fixed point near  $x = 1.3$ .
- The orbit is periodic with period two.
- The orbit appears to be aperiodic.

### Question 3

The bifurcation diagram for an iterated function (not the logistic equation) is shown in Question 1. Which statement best describes the long-term behavior of orbits of this dynamical system if  $r=1.9$ ?

- The orbit approaches zero.
- The orbit approaches a fixed point at  $x = 1$ .
- The orbit approaches a fixed point near  $x = 1.3$ .
- The orbit is periodic with period two.
- The orbit appears to be aperiodic.

#### Question 4

Answer this question by using the web program that makes bifurcation diagrams for the logistic equation. Near  $r = 3.740$  there is a period-five window. A bifurcation from period 5 to period 10 occurs near what  $r$  value?

- $r = 3.738$
- $r = 3.741$
- $r = 3.743$
- $r = 3.745$

#### Question 5

Answer this question by using the web program that makes bifurcation diagrams for the logistic equation. Near  $r = 3.702$  there is a periodic window of what period?

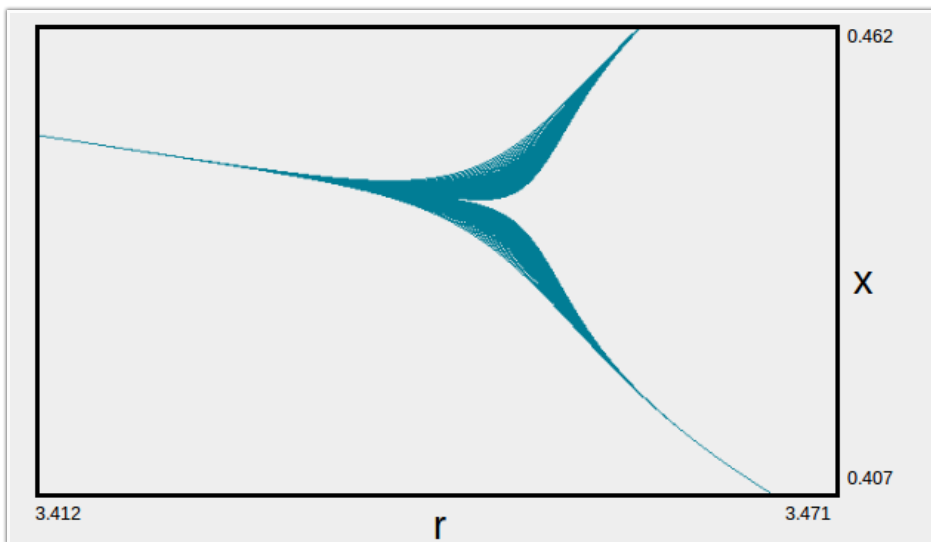
- 3
- 4
- 5
- 6
- 7

#### Question 6

Answer this question by using the web program that makes bifurcation diagrams for the logistic equation. Near  $r = 3.582$  there is a periodic window of what period?

- 4
- 5
- 10
- 12
- 20

#### Question 7



Near a bifurcation, the bifurcation diagram often becomes "fuzzy" or blurry, as shown above. (This is a zoomed-in view of the transition from period two to period four.) What is the cause of this blurring?

- The program is not plotting enough points.
- The attractor is very weakly attracting, so it takes the orbit a very long time to reach the attractor.
- Sensitive dependence on initial conditions.
- The shadowing lemma

#### Question 8

Near a bifurcation, the bifurcation diagram often becomes "fuzzy" or blurry, as shown above. (This is a zoomed-in view of the transition from period two to period four.) To make the bifurcation diagram appear sharper, what should one do?

- Make the program plot more iterates
- Make the program skip more iterates before it starts plotting.