

## Instructions

Note: If the equations in this test do not display properly, please download the pdf version of this test [from this link](#). This file will show the equations correctly. Do not click on "Download Exam PDF". We apologize for the inconvenience.

You may use any course materials, videos, websites, calculators, etc. for this test. Just don't ask another person for the answers or share your answers with other people. Please do not post questions about the test on the forum. If you have questions, please send them via email to [chaos@complexityexplorer.org](mailto:chaos@complexityexplorer.org). Thanks.

### Question 1

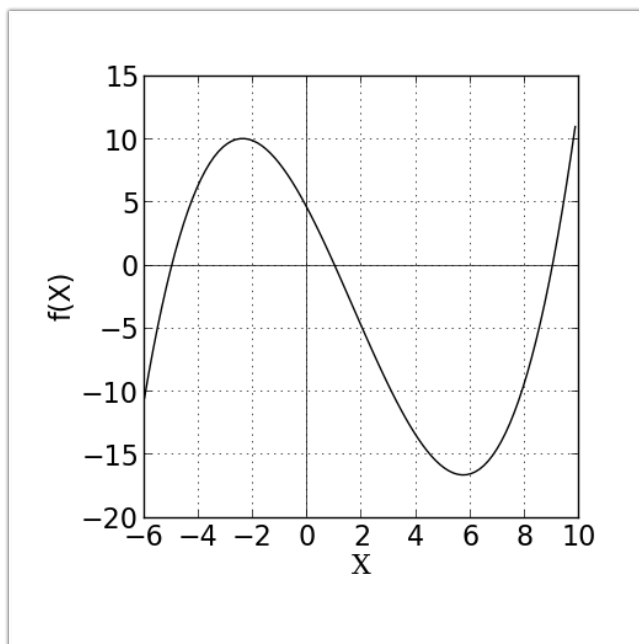
Consider the differential equation:

$$\frac{dT}{dt} = 0.2(20 - T)$$

Let the initial temperature be 5 degrees. (That is,  $T(0) = 5$ .) Using Euler's method with  $\Delta t = 0.5$ , what does one obtain for  $T(0.5)$ , the temperature when the time  $t = 0.5$ ?

- A. 6
- B. 6.5
- C. 9
- D. 11

### Question 2



Consider the differential equation  $\frac{dx}{dt} = f(x)$ , where  $f(x)$  is plotted above. Which of the following statements is true about this differential equation?

- A. It has an unstable fixed point at -5, a stable fixed point at 1, and an unstable fixed point at 9
- B. It has an unstable fixed point near -2.3 and a stable fixed point near 5.7
- C. It has a stable fixed point at -5, an unstable fixed point at 1, and a stable fixed point at 9
- D. It has stable fixed points at -5, 1, and 9
- E. It has unstable fixed points near -2.3 and 5.7

### Question 3

Consider the differential equation  $\frac{dx}{dt} = f(x)$ , where the graph of  $f(x)$  is shown in the statement of question 2, above. What is the long-term behavior of a

solution to this differential equation that starts at  $X=4$ ? (I.e.,  $X(0)=4$ .)

- A. It increases and approaches around 5.8
- B. It increases and approaches 9
- C. It oscillates between 4 and 7
- D. It decreases and approaches 1
- E. It decreases and tends toward negative infinity

#### Question 4

Consider the differential equation  $dX/dt = f(X)$ , where the graph of  $f(X)$  is shown in the statement of question 2, above. What is the long-term behavior of a solution to this differential equation that starts at  $X=0$ ? (I.e.,  $X(0)=0$ .)

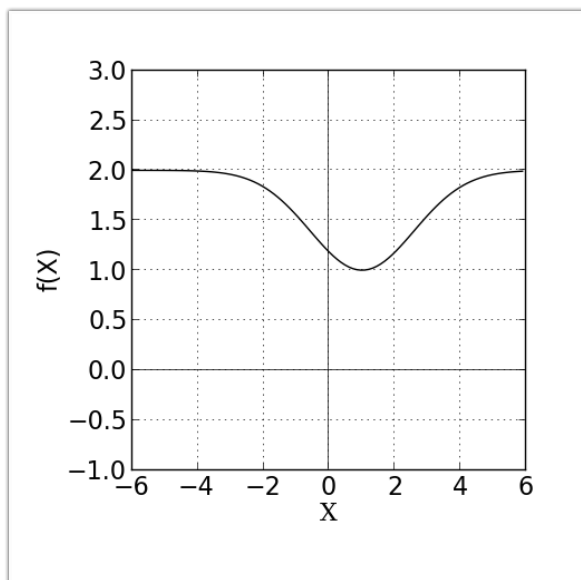
- A.  $X(t)$  increases and approaches 5.7
- B.  $X(t)$  does not change. 0 is a fixed point
- C.  $X(t)$  increases and tends toward infinity
- D.  $X(t)$  increases and approaches 1
- E.  $X(t)$  decreases and approaches -5

#### Question 5

Consider the differential equation  $dX/dt = f(X)$ , where the graph of  $f(X)$  is shown in the statement of question 2, above. What is the long-term behavior of a solution to this differential equation that starts at  $X=1$ ? (I.e.,  $X(0)=1$ .)

- A.  $X(t)$  increases and approaches  $X = 5.7$
- B.  $X(t)$  does not change. 1 is a fixed point.
- C.  $X(t)$  increases and approaches 9
- D.  $X(t)$  decreases and approaches -3
- E.  $X(t)$  decreases and approaches -5

#### Question 6



Consider the differential equation  $dX/dt = f(X)$ , where  $f(X)$  is shown in the graph above. What is the long-term behavior of a solution to this differential

equation that starts at  $X=-2$ ? (That is,  $X(0) = -2$ .)

- A.  $X(t)$  tends toward negative infinity
- B.  $X(t)$  increases and approaches 0
- C.  $X(t)$  increases and approaches 1
- D.  $X(t)$  tends toward positive infinity

### Question 7

Consider the differential equation  $dX/dt = f(X)$ , where  $f(X)$  is shown in the statement of question 6. Which of the following statements is true about this differential equation?

- A. It has a stable fixed point at  $X=1$
- B. It has an unstable fixed point at  $X=1$
- C. It has no fixed points

### Question 8

True or False: From a phase line for a differential equation one can determine exact, numerical values for all solutions.

- A. True
- B. False