

Introduction to Dynamical Systems and Chaos (Winter 2014)

7.8 Test » Unit 7 Test

Instructions 1

You may use any course materials, videos, websites, calculators, etc. for this test. Just don't ask another person for the answers or share answers with other people. Please do not post questions about the test on the forum. If you have questions, please send them via email to chaos@complexityexplorer.org. Thanks.

Question 2

Can two-dimensional differential equations of this form:

$$\frac{dx}{dt} = f(x, y)$$

$$\frac{dy}{dt} = g(x, y)$$

exhibit chaotic behavior? Assume that $f(x, y)$ and $g(x, y)$ are smooth and continuous functions.

- ☐ Yes
 - ☐ No
-

Question 3

$$\frac{dx}{dt} = f(x, y, z)$$

$$\frac{dy}{dt} = g(x, y, z)$$

$$\frac{dz}{dt} = h(x, y, z)$$

Can three-dimensional differential equations of the form shown above exhibit chaos? Assume that f , g , and h are all smooth and continuous functions.

- ☐ Yes
 - ☐ No
-

Question 4

$$\frac{dx}{dt} = f(x, y)$$

$$\frac{dy}{dt} = g(x, y)$$

Consider a set of coupled differential equations as shown above. (As usual, $f(x, y)$ and $g(x, y)$ are smooth and continuous functions.)

If solution curves $x(t)$ and $y(t)$ for two different initial conditions are plotted in phase space, is it possible for those curves to intersect?

- ☐ Yes
- ☐ No

Question 5

$$x_{n+1} = 1 + y_n - ax_n^2$$

$$y_{n+1} = bx_n$$

Consider the Henon map, shown above. Let $a=0.7$ and $b=0.2$. If $x_0 = 0.3$ and $y_0 = 0.2$, what are x_1 and y_1 ?

- ☐ $x_1 = 0.99, y_1 = 0.02$
 - ☐ $x_1 = 0.99, y_1 = 0.04$
 - ☐ $x_1 = 1.263, y_1 = 0.06$
 - ☐ $x_1 = 1.137, y_1 = 0.06$
-

Question 6

$$x_{n+1} = 1 + y_n - ax_n^2$$

$$y_{n+1} = bx_n$$

Consider the Henon map, shown above. Let $a=0.7$ and $b=0.2$. Let $x_0 = 0.3$ and $y_0 = 0.2$. What is the long-term behavior of the orbit for initial conditions?

- ☐ The orbit approaches a fixed point.
 - ☐ The orbit approaches a cycle of period two.
 - ☐ The orbit approaches a cycle of period four.
 - ☐ The orbit appears to be aperiodic.
-

Question 7

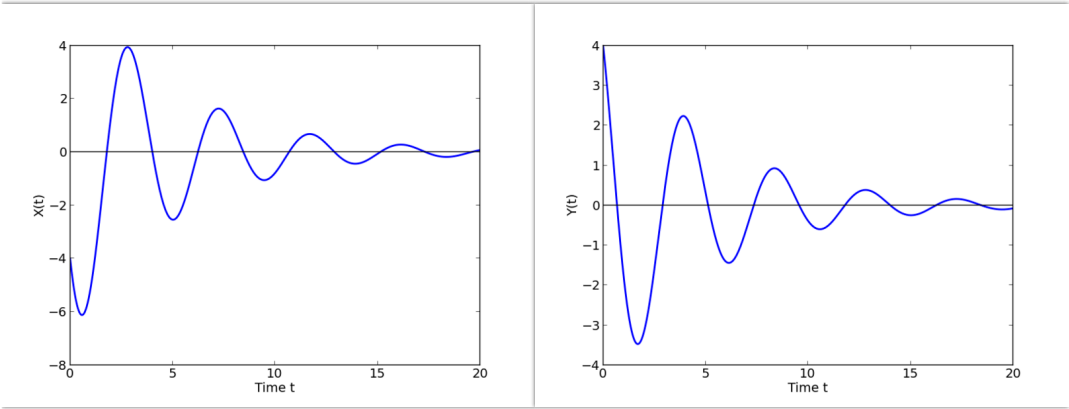
$$x_{n+1} = 1 + y_n - ax_n^2$$

$$y_{n+1} = bx_n$$

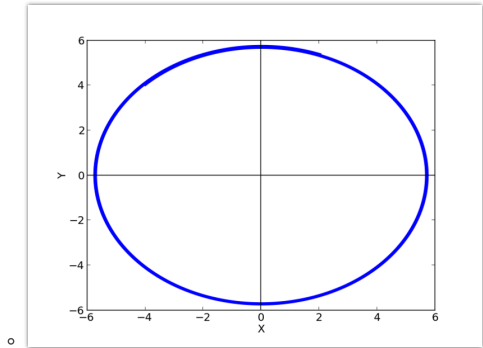
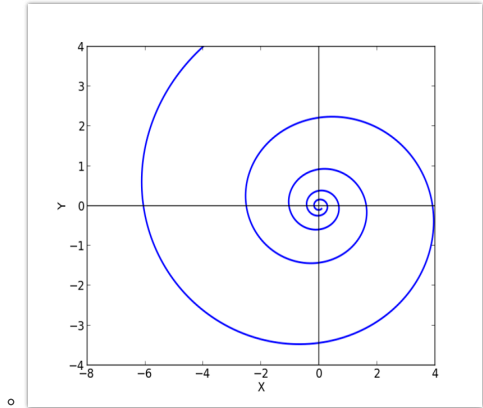
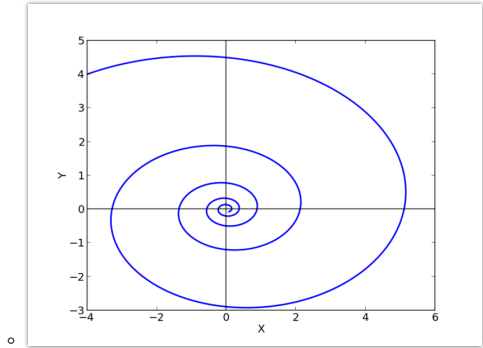
Consider the Henon map, shown above. Let $a=1.3$ and $b=0.2$. Let $x_0 = 0.5$ and $y_0 = 0.2$. What is the long-term behavior of the orbit for condition?

- ☐ The orbit approaches a fixed point.
- ☐ The orbit approaches a cycle of period two.
- ☐ The orbit approaches a cycle of period four.
- ☐ The orbit appears to be aperiodic.

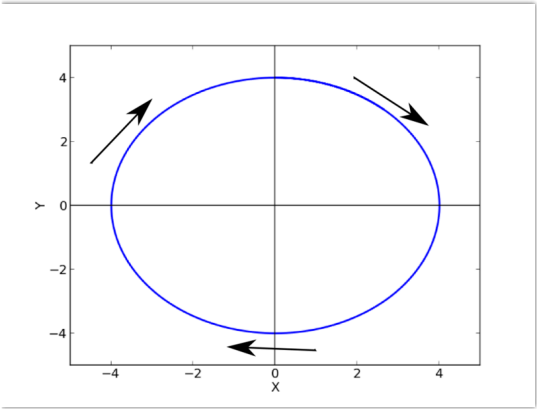
Question 8



The solutions $x(t)$ and $y(t)$ to a differential equation are shown above. What would these solutions look like if they were plotted in the plane?



Question 9



The solution to a differential equation is shown above in the phase plane. The direction of motion is clockwise, as indicated by the arrows. The direction of motion is clockwise, as indicated by the arrows of the following solutions $x(t)$ and $y(t)$ would yield the phase space plot shown above?