

## Instructions

You may use any course materials, websites, books, computer programs, calculators, etc. for this test. Just don't ask another person for the answers or share your answers with other people. Be aware that simply typing the question text into google is unlikely to get you directly to the right answer; you're going to have to read what you find there in order to extract that answer, and the course videos are probably a faster way to do that.

"Experts" notes clarify situations that haven't been covered in this course, but that may introduce subtleties into the exam answers. Do not worry about them unless you understand the terms and issues in those notes.

If you have questions about this test, please email us at [nonlinear@complexityexplorer.org](mailto:nonlinear@complexityexplorer.org) rather than posting on the forum.

### Question 1

Measuring one state variable from a dynamical system effectively projects that system's dynamics onto a line.

- True
- False

### Question 2

If a sensor does not measure a state variable of a dynamical system directly, but rather measures the *product* of two state variables, you can't use delay-coordinate embedding to reconstruct the dynamics from time-series data measured by that sensor.

- True
- False

### Question 3

This question, and the seven that follow, concern the topological equivalence between the true and reconstructed dynamics that is assured by the delay-coordinate embedding theorems.

That equivalence is useful because many dynamical invariants — important quantities like the Lyapunov exponent — are invariant under diffeomorphism.

- True
- False

### Question 4

The equivalence noted in the header of question 3 is always apparent from *visual* examination of the reconstructed trajectory.

- True
- False

### Question 5

The equivalence noted in the header of question 3 means that there aren't any trajectory crossings in the reconstructed dynamics. (Experts: neglect nonautonomous systems.)

- True
- False

### Question 6

According to the original (Takens) theorems, the equivalence noted in the header of question 3 exists if the embedding dimension is at least as high as the dimension of the true dynamics.

- True
- False

### Question 7

In theory, any delay  $\tau > 0$  (and not a multiple of any orbit period in the system under study) will satisfy the equivalence noted in the header of question 3.

- True
- False

### Question 8

In theory **and in practice**, any delay  $\tau > 0$  (and not a multiple of any orbit period in the system under study) will satisfy the equivalence noted in the header of question 3.

- True
- False

### Question 9

The equivalence noted in the header of question 3 requires that the time series is measured by a sensor that effects a smooth (and generic) function of at least one state variable of the dynamical system.

- True
- False

### Question 10

In practice, the equivalence noted in the header of question 3 can depend on...

- The length of the time series.
- How much noise is in the data.
- Whether the signal is stationary or nonstationary.
- All of the above.
- None of the above.
- Some but not all of the above.

### Question 11

Here's some data:

x    time

1.2  1

1.4  2

1.1  3

0.9  4

0.5  5

0.1  6

-0.2 7

0.3  8

0.4  9

If you were to perform a delay-coordinate embedding of that time-series data with  $m=2$  and  $\tau=2$ , what would the third point be?

- (1.2, 1.1)
- (1.2, 1.1, 0.5)
- (1.1, 0.5)
- (0.9, 0.1)
- (0.5, -0.2, 0.4)

#### Question 12

If you were to perform a delay-coordinate embedding of the time-series data in question 11 with  $m=3$  and  $\tau=1$ , what would the second point be?

- (1.4, 1.1, 0.9)
- (1.1, 0.5, -0.2)
- (1.2, 1.4)
- (1.1, 0.9, 0.5)
- (1.4, 1.1)

#### Question 13

Noise is a problem for the false-near neighbor technique because...

- It makes the algorithm run more slowly.
- It alters neighbor relationships.
- It alters the time scales of the signal.

#### Question 14

How might you be able to tell if your time series is nonstationary?

- By downsampling the data (i.e., throwing out two out of every three points) and seeing if the results change.
- By performing the same nonlinear time-series analysis procedures on chunks of the data (e.g., first half and second half) and seeing if the results are different.
- By visiting the oracle of Delphi.