Instructions

You may use any course materials, websites, books, computer programs, calculators, etc. for this test. Just don't ask another person for the answers or share your answers with other people. Be aware that simply typing the question text into google is unlikely to get you directly to the right answer; you're going to have to read what you find there in order to extract that answer, and the course videos are probably a faster way to do that.

"Experts" notes clarify situations that haven't been covered in this course, but that may introduce subtleties into the exam answers. Do not worry about them unless you understand the terms and issues in those notes.

If you have questions about this test, please email us at nonlinear@complexityexplorer.org rather than posting on the forum.

Question 1

Why is it useful to transform an nth-order ODE into n 1st-order ODEs?

- A. It's the only way to know how many state variables are involved.
- B. To see if it's linear.
- C. Because that's the form in which ODE solvers want them.
- D. Because it's fun.

Question 2

If you transformed this ODE into a set of 1st-order ODEs, how many "helper variables" would you need?

x''' + x' - 3 sinx = 0

- A. None.
- B. This transformation isn't possible for this ODE.
- C. One
- D. Two
- E. Three

Question 3

What is the state vector of the simple harmonic oscillator (SHO)?

- $[x, v]$
- $[x, v]^T$
- $[\theta, \omega]$

Question 4

The SHO equations are below:

$x' = v$

$v' = -x$

If x=1 and v=1, what are x' and v'?

- x'=1 and v'=0
- x'=-1 and v'=1
- x'=1 and v'=-1
- None of the above

Question 5

Refer to to the SHO equations in question 4.

If x=0 and v=1, what are x' and v'?

- x'=0 and v'=1
- x'=1 and v'=0
- x'=0 and v'=0
- None of the above

Question 6

The state-space point x=0, v=0 is a fixed point of the SHO dynamics.

- True
- False

Question 7

The state-space point x=1, v=0 is a fixed point of the SHO dynamics.

- True
- False

Question 8

Difference equations and differential equations can both involve multiple state variables.

- True
- False

Question 9

Difference equations and differential equations both involve derivatives.

- True
- False

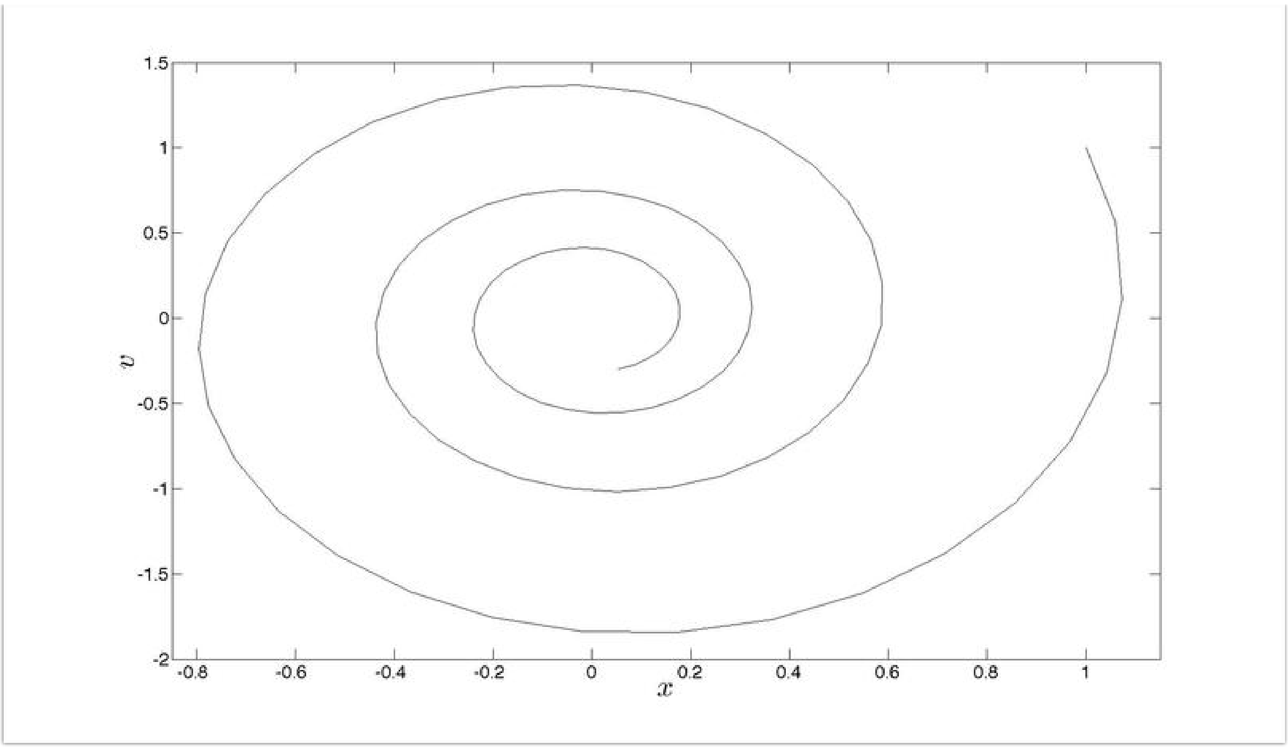
Question 10

A difference equation gives you the next state of the system,whereas a differential equation gives you the direction in which the state evolves.

- True
- False

Question 11

This picture shows a trajectory of the undamped SHO system, starting from [1,1] with a step size of 0.1. Which solver was used to generate it?



- forward (explicit) Euler
- backward (implicit) Euler

Question 12

What will happen to the solutions generated by forward and backward Euler methods as you change the time step? (experts: neglect the effects of floating-point arithmetic when you're answering this question.)

- Solutions produced by both methods will improve (i.e., get more accurate) as you decrease the time step.
- Solutions produced by both methods will improve (i.e., get more accurate) as you increase the time step.
- There will be no change in accuracy in either method as you decrease the time step.

Question 13

Comparing the amount of work—the number of operations—done by a computer that is running the forward Euler algorithm to generate a one-second-long trajectory of the SHO system using two different time steps: 0.1 and 0.2 seconds. (Experts: only consider main loop iteration costs, not startup, and neglect all floating-point effects.)

- Both trajectories will require about the same amount of computational effort.
- The trajectory with the longer time step will require about twice as much work.
- The trajectory with the shorter time step will require about twice as much work.
- The trajectory with the shorter time step will require more than twice as much work.
- The trajectory with the longer time step will require more than twice as much work.

Question 14

Use your forward Euler solver from HW 5.4 on the SHO equations with k=2, m=1, and β=0, from the initial condition x(t=0)=-1 , v(t=0)=-2, with a timestep of 0.05, to compute the values of x and v at t=0.5. [Note: this is problem 1 on HW 5.4 with a different m and a different timestep. Also note that we use the symbols "h" and "delta t" interchangeably to mean "timestep."]

- $x(t=0.5) \approx -1.528,\ v(t=0.5) \approx 0.625$
- $x(t=0.5) \approx -1.7209,\ v(t=0.5) \approx -0.6198$

- $\boxed{x(t = 0.5) \approx -1.7621,\ v(t = 0.5) \approx -0.6439}$